#1666 : Autonomous Exploration for Navigating in MDPs using Blackbox RL Algorithms

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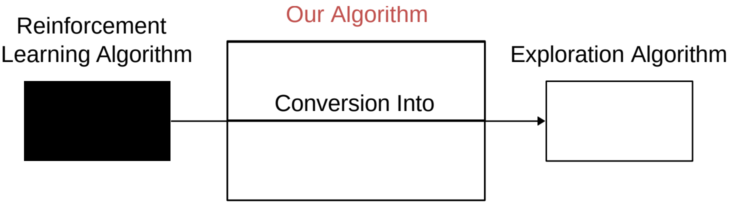
- Our work :



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  - Starting state s_0.
- Assumption : In every state, RESET action available which leads back to s_0.
- Input : $L \geq 1$.
  Goal : Find a policy for every state reachable from the starting state s_0 in L steps.

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Navigation time$_\pi(s)$

Expected #steps before reaching state s for the first time following policy π from the starting state s_0.

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- Goal : Find a policy $\forall\, s \in \mathcal{S}_L^{\rightarrow}$ with navigation time $\leq (1+\epsilon)L$.

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- META-EXPLORE proceeds in *rounds*.
  In each round, it evaluates a *target state*.
- Target states are chosen from the set of candidate states.
- If $(1+\epsilon)L$-step policy found for the target state ,
  Successful round and target state becomes *known*.
  Else
  Failure round.

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- Exploring the neighborhood of known states to add to the set of candidate states $\mathcal{U}$.
- In a newly known state, every action is sampled $\tilde{O}(L)$ times.
- Any newly discovered states
  and the neighboring states of previously known states
  are added to the set of candidate states $\mathcal{U}$.

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- Chosen arbitrarily from the set of candidate states.
- Algorithm stops when the set of candidate states is empty.

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An MDP such that regret minimization leads to time-effective navigating to the target state.

- Induced MDP : In the induced MDP $\mathcal{M}_{\bar{s}}$ for target state $\bar{s}$, the learner

  has loss 0 in $\bar{s}$, and
  suffers loss 1 in every other state.

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- At the end of a successful round, $\mathcal{K} = \mathcal{K} + \bar{s}$ and all associated history points are added to the output for $\bar{s}$.

Navigation Policy for Known States

For each known state $s \in \mathcal{K}$,

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3. If s is not reached in $\approx \frac{L}{\epsilon}$ steps, RESET and go to step 1.

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3. *for each* $s \in \mathcal{K}$*, outputs a policy with navigation time* $\leq (1 + \epsilon)L$.

Concluding Remarks

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Thank You.

Scan the following to see the paper

See you at the poster D1.

References

[LA12] Shiau Hong Lim and Peter Auer. "Autonomous Exploration For Navigating In MDPs". In: *Proceedings of the 25th Annual Conference on Learning Theory*. 2012, pp. 40.1–40.24.

[TPVL20] Jean Tarbouriech et al. "Improved Sample Complexity for Incremental Autonomous Exploration in MDPs". In: *Advances in Neural Information Processing Systems*. 2020, pp. 11273–11284.

Diameter of an MDP

Consider the stochastic process defined by a stationary policy $\pi : \mathcal{S} \rightarrow \mathcal{A}$ operating on an MDP M with initial state s_0. Let $T(s'|M, \pi, s)$ be the random variable for the first time step in which state s' is reached in this process. Then the diameter of M is defined as

$$D(M) := \max_{s \neq s'} \min_{\pi:\mathcal{S}\rightarrow\mathcal{A}} \mathbb{E}\left[T(s'|M, \pi, s)\right]$$

Incrementally Reachable States : Definition

Definition (Incrementally reachable states)

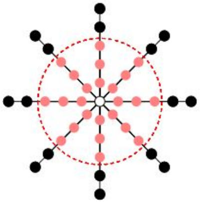
Let $\prec$ be some partial order on $\mathcal{S}$. The set $\mathcal{S}_L^{\prec}$ of states reachable in L steps with respect to $\prec$, is defined inductively as follows:

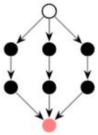
- $s_0 \in \mathcal{S}_L^{\prec}$,
- if there is a policy π on $\{s' \in \mathcal{S}_L^{\prec} : s' \prec s\}$ with navigation time$\pi(s)$ $\leq L$, then $s \in \mathcal{S}_L^{\prec}$.

We define the set $\mathcal{S}_L^{\rightarrow}$ of states incrementally reachable in L steps with respect to some partial order to be $\mathcal{S}_L^{\rightarrow} := \bigcup_{\prec} \mathcal{S}_L^{\prec}$, where the union is over all possible partial orders.

Incrementally Reachable States : Illustration

- Two environments where the starting state s_0 is shown in white.
- On the left, each transition is deterministic and is depicted with an edge.
- On the right, each transition from s_0 to the first layer is equiprobable, and the rest of the transitions are deterministic.
- For $L = 3$, states belonging to S_L are shown in pink.
- On the left, $\mathcal{S}_L^{\rightarrow} = \mathcal{S}_L$. On the right, $\mathcal{S}_L^{\rightarrow} = \{s_o\} \neq \mathcal{S}_L$.





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- actions in all the other states behave the same as in the original MDP and suffer loss 1.

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If META-EXPLORE *is run with an online RL algorithm* $\mathfrak{A}$ *with a regret upper bound of* $B(\#\mathit{States}, \#\mathit{Actions}) \cdot T^{\alpha} \cdot D^{\beta}$, *then with prob.* $1 - \delta$, *it*

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2. *terminates after*

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3. *for each* $s \in \mathcal{K}$, *outputs a policy with navigation time* $\leq (1+\epsilon)L$.

Relation to Existing Work

UCBEXPLORE [LA12]	$\tilde{O}(SAL^3/\epsilon^3)$
DISCO [TPVL20]	$\tilde{O}(SAGL^3/\epsilon^2)$
META-EXPLORE using UCRL2b	$\tilde{O}(S^3GA^2L^2/\epsilon^4)$