Introduction to Reinforcement Learning

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2AMM20 Research Topics in Data Mining
Eindhoven University of Technology

Preliminaries

Objectives

- To gain an understanding of various reinforcement learning problems and formulate them mathematically.
- To devise solution strategies for these problems.
- To prove performance guarantees for these solutions.

Prerequisites

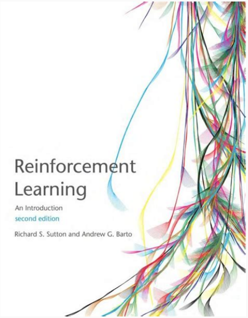
- Elementary statistics and probability theory.
- Comfort with applying mathematical tools.
- Bachelor's course worth of background knowledge in Data Mining and Machine Learning.

Class Information

- Course webpage : https://canvas.tue.nl/courses/21915/pages/reinforcement-learning-track-page
- Uploaded lecture slides may be updated as the course progresses.
- Contact me : p.gajane@tue.nl
- Please put [2AMM20] (with the square brackets) in the subject line of your email.

Resources

- Reinforcement Learning – An Introduction [Sutton and Barto, 2018][Chapter 1, 2 and 3]
- Bandit Algorithms [Lattimore and Szepesvari, 2020]
- Markov Decision Processes: Discrete Stochastic Dynamic Programming [Puterman, 1994][Chapter 4]
- Research Articles



Lecture 1 : Outline

- What is Reinforcement Learning?
- Elements of a Reinforcement Learning (RL) Problem
- Formulating RL with Multi-Armed Bandits
- Formulating RL with Markov Decision Processes

What is Reinforcement Learning?

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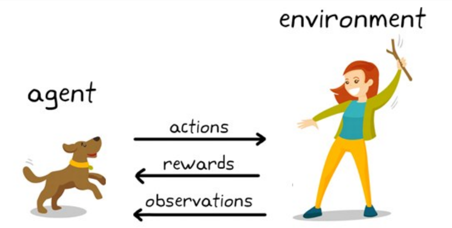
- Learning : Learning occurs when performance at given tasks improves with experience [Mitchell, 1997].
- Definition not all encompassing : Breaking-in a new pair of shoes. Do the shoes *learn* to fit our feet better?
- How do people and animals learn?

Learning by Reinforcement



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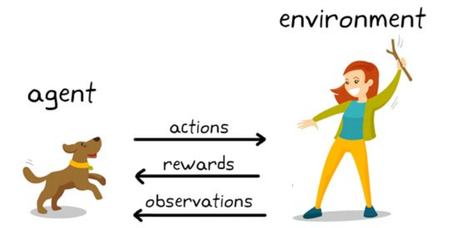

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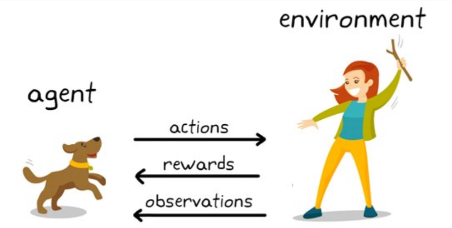

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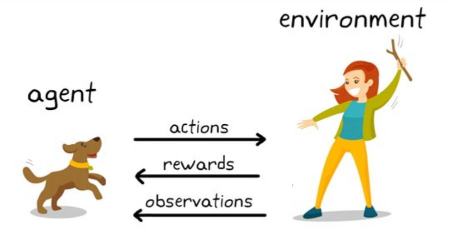

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- The dog performs an action.
- If desired action,

  then reward,

  otherwise

  no (or negative) reward.

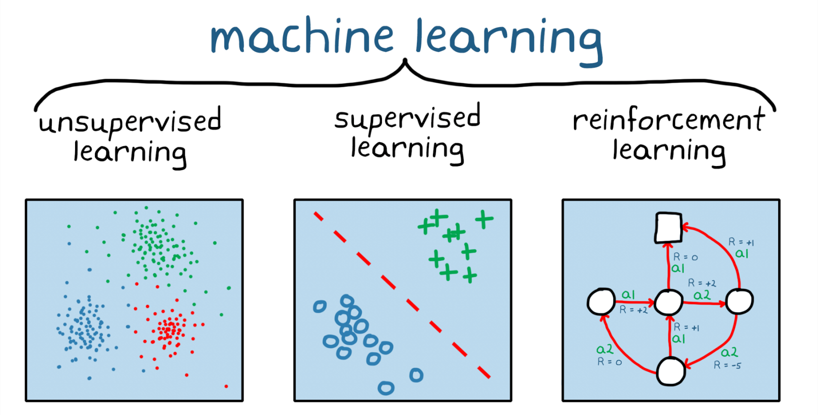
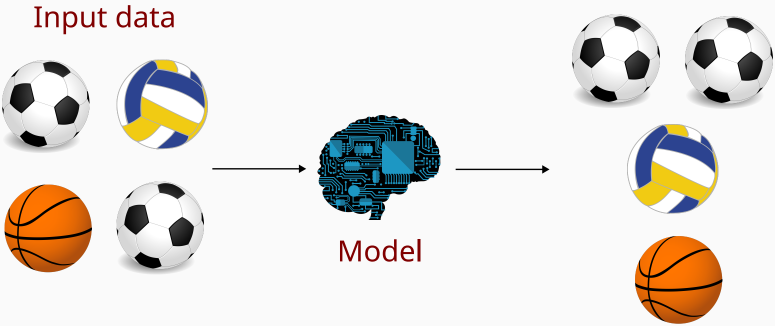


Image source: *mathworks*

Figure 1: Basic machine learning paradigms.

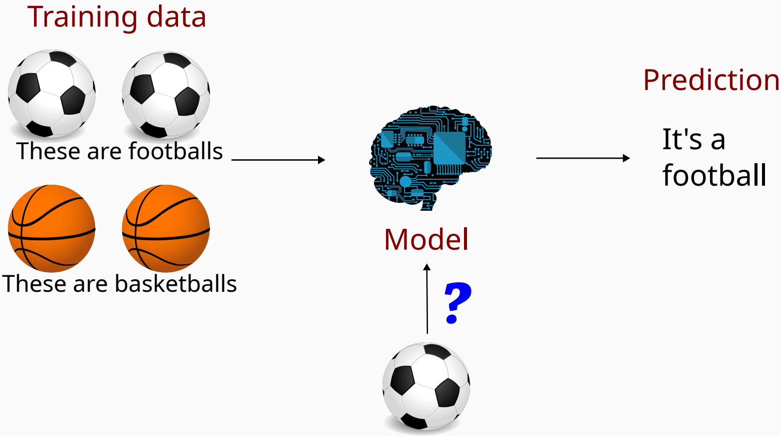
Unsupervised Learning

Aims to find structures/clusters in unlabeled data.



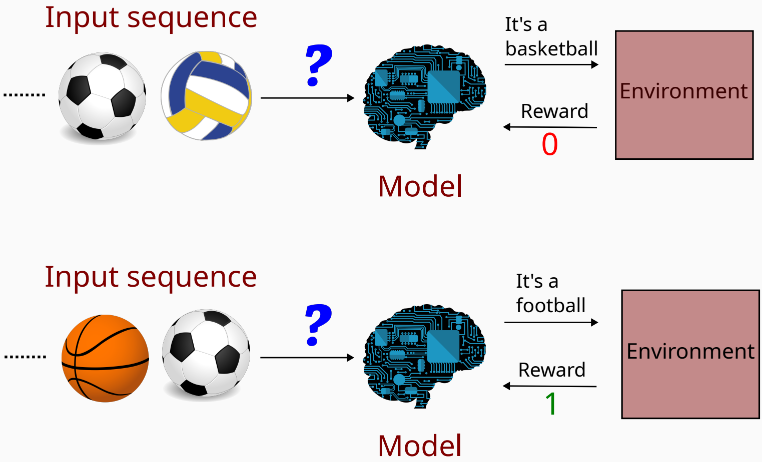
Supervised Learning

Learning from training data of labeled examples provided by a knowledgeable external supervisor.



Reinforcement Learning

Learning from the feedback provided by the environment in response to the model's behavior to optimize the reward.



Examples of Reinforcement Learning

- Make a humanoid robot walk.

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- Play many different Atari games.
- Ad placement.
- Fly stunt manoeuvres in a helicopter.

Features of Reinforcement Learning

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- Interactions are often sequential.
- It is active, rather than passive.

Distinguishing Factors of Reinforcement Learning

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  - No external supervisor, only a reward signal.
  - No need to obtain representative and correct training samples.

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- Exploration/exploitation dilemma.

Exploration/Exploitation Dilemma

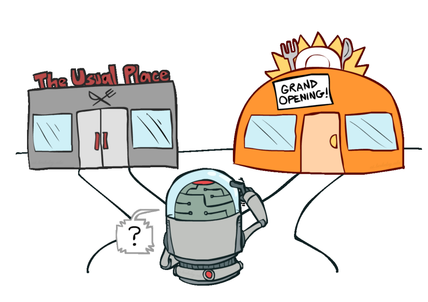

Image source: *UC Berkeley AI course, lecture 11*

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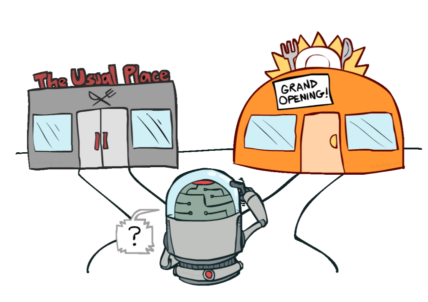

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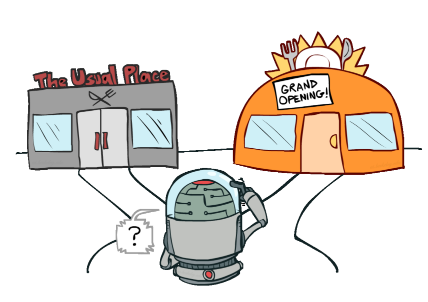

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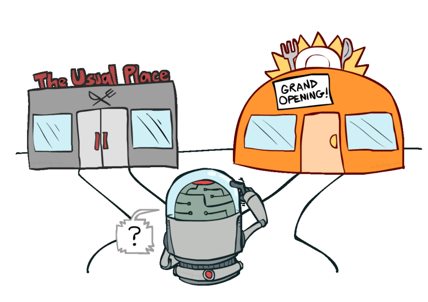

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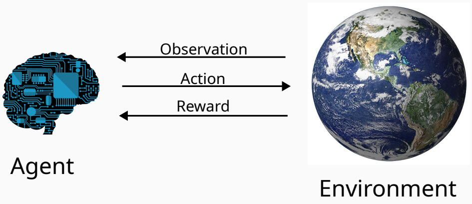
- Exploit. Choose actions tried in the past and found to be rewarding.
- Explore. Choose unexplored actions to see if they are more rewarding.
- Neither exploration nor exploitation can be pursued exclusively.

Reinforcement Learning : Problem Formulation

Reinforcement Learning : Agent and Environment



- Emits observation $o(t)$.

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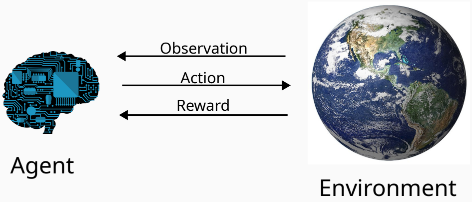

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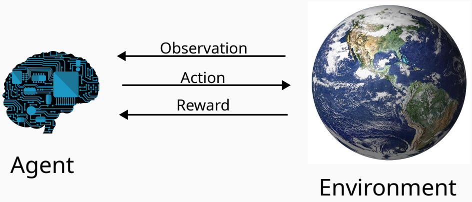

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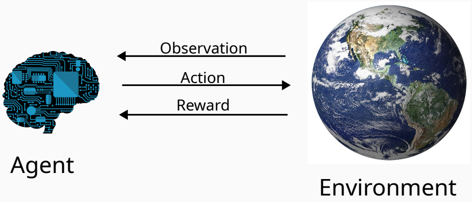

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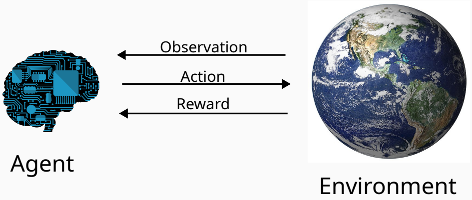

- Receives observation $o(t)$.
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- Emits observation $o(t)$.
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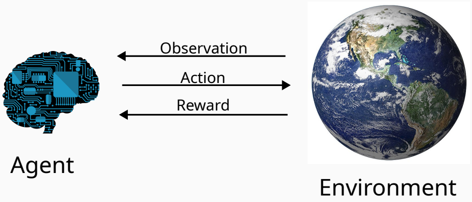

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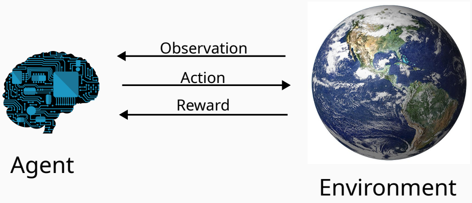

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- Horizon T : time step when the process ends.

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- Model : Interpretation of the environment's behaviour.

Elements of a RL Problem : State and Action

Image source: *Chess.com*

- State $s \in \mathcal{S}$ describes the current situation.
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- Actions $a \in \mathcal{A}$ are the choices available to the agent.
- Actions are permitted to affect the future state.

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- Reward hypothesis : *Any goal can be formalized as the outcome of maximizing a cumulative reward.*

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  - Ad placement : +ve reward for every click,
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- Being greedy might not work : sometimes better to sacrifice short term reward to gain more long-term reward.

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- Can be used to evaluate desirability of states and choose between actions.

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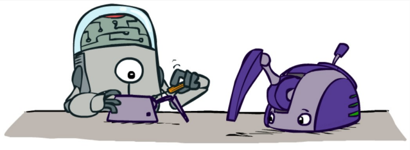

Figure 2: Model-based learning

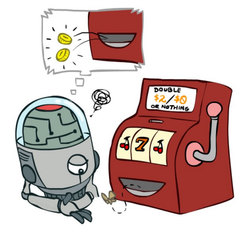

Figure 3: Model-free learning

An Example of a RL Problem I

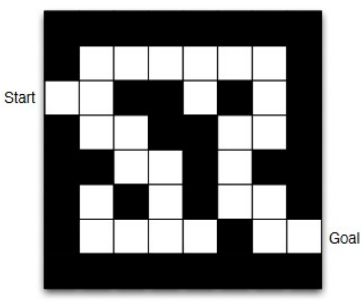

Image source: *DeepMind RL course*

- Move from start state to goal state as quickly as possible.

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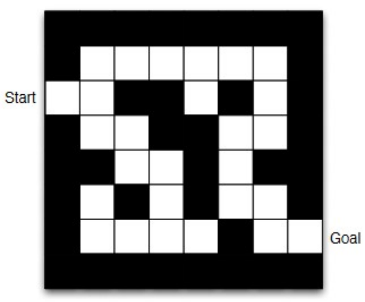

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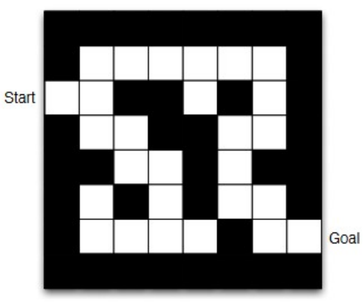

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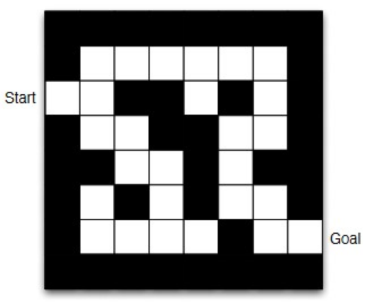

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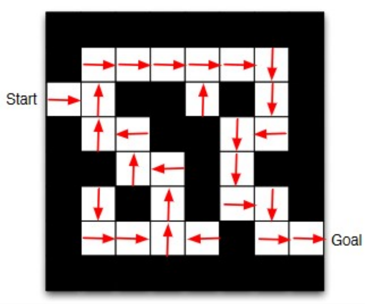
- State : agent's location.
- Actions : $\uparrow$, $\downarrow$, $\leftarrow$, $\rightarrow$.

An Example of a RL Problem II



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- Arrows represent policy $\pi(s)$ for each state s.

An Example of a RL Problem III

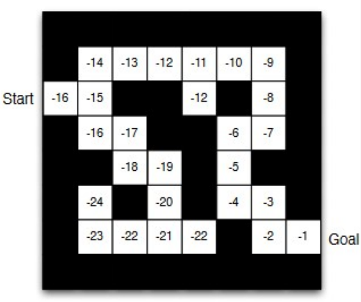

Image source: *DeepMind RL course*

- Numbers represent values $v_\pi(s)$ of each state s.

Break

We start again after a break.

Measuring the Performance: Optimal Value Function

- Recall that,

  undiscounted value for policy π is,

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Definition

The optimal value function $v_*(s) = max_\pi v_\pi(s)$.

- The optimal value function specifies the best possible performance.

Measuring the Performance: Optimal Policy

- There exists an optimal policy π_* that is better than or equal to all other policies[1].

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where $\pi_1 \geq \pi_2$ if $v_{\pi_1}(s) \geq v_{\pi_2}(s), \forall s$

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- A RL problem is "solved" when the agent finds an optimal policy.

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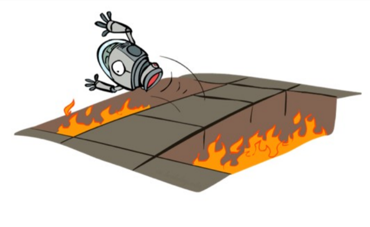
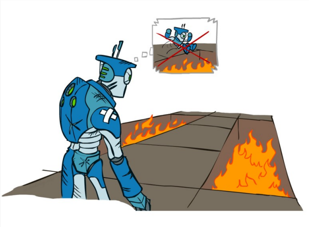
Measuring the Performance : Regret





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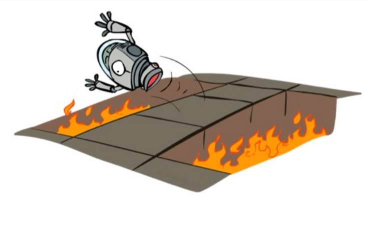
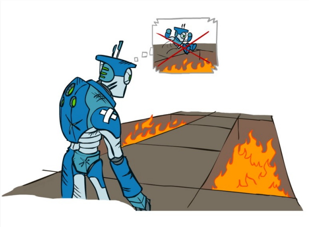

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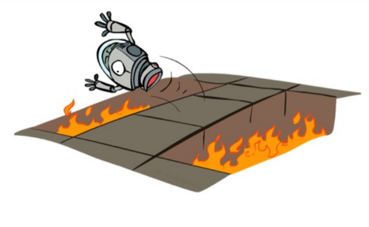

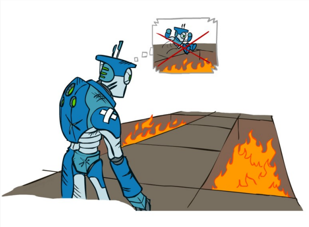

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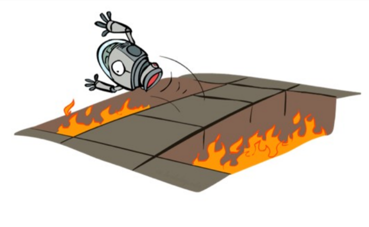

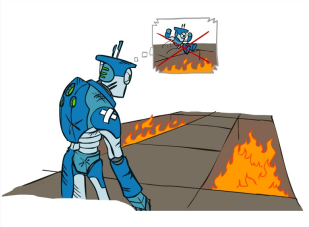

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- Minimizing regret equivalent to maximizing cumulative reward.

Non-associative RL : Multi-Armed Bandits

Multi-armed bandits

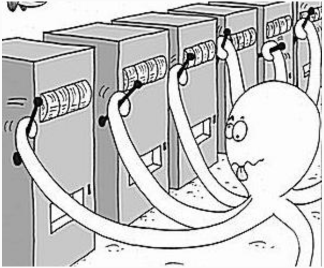

Image source: *Microsoft research*

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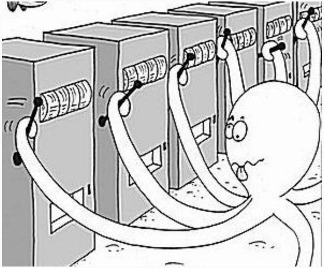
Multi-armed bandits



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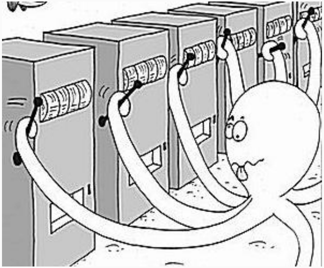

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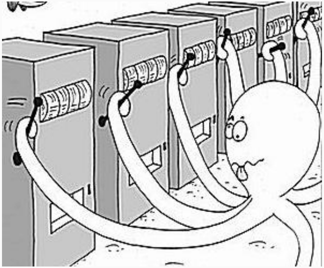

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- Learning to act in a single situation i.e. state.
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- After each choice, the learner receives a numerical reward.
- Goal : Maximize the cumulative reward or minimize the regret.

Stationary Stochastic Bandits

- Reward for arm a drawn i.i.d. from an unknown stationary distribution.

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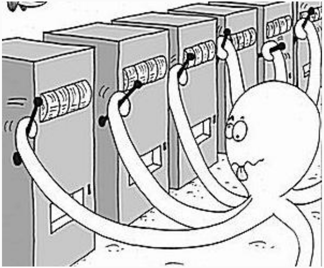
- A variant: Non-stationary stochastic bandits - rewards are drawn from distributions which may change over time.

Adversarial Bandits



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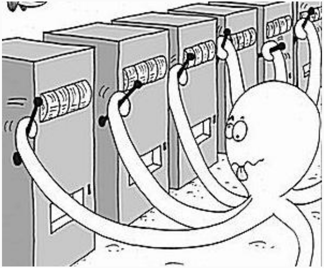

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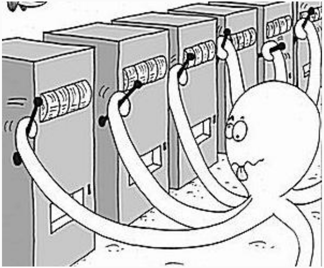

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- The assumption of stationary stochastic distributions is optimistic and sometimes unrealistic.
- Pessimistic assumption : rewards are chosen adversarially.
- Oblivious adversary : rewards for all arms and all rounds are chosen in advance.

Dueling Bandits

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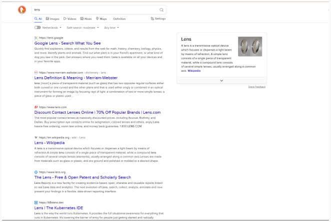

Figure 4: DuckDuckGo search results

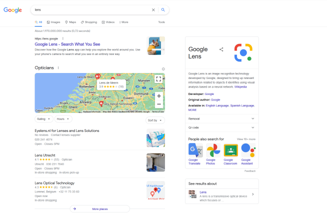

Figure 5: Google search results

- Practical scenario : Information retrieval in search engines.
- Relative feedback by interleaved filtering [Radlinski and Joachims, 2007]

Contextual Bandits

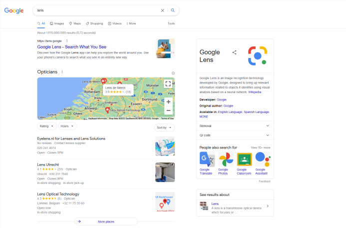

Figure 6: Google search results

- Observation of extra information (*context*) before choosing an action.
- Practical scenario : News recommendation, ad selection.

Associative RL: Markov Decision Processes

History and State

- History is the sequence of observations, actions and rewards.

$$\mathcal{F}_t = o(1), r(1), a(1), \ldots, a(t-1), o(t), r(t).$$

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    - Agent selects an action $a(t)$.
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- State is the information used to determine what happens next.
- Formally, state is a function of the history: $s(t) = f(\mathcal{F}_t)$.

Markov Property

- "The future is independent of the past given the present".

Andrey Markov(1856-1922)

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- The state $s(t)$ is Markov if and only if

$$\mathbb{P}(s(t+1) \,|\, s(t)) = \mathbb{P}(s(t+1) \,|\, s(t), s(t-1), \ldots, s(1)).$$

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$$\mathbb{P}(s(t+1) \mid s(t)) = \mathbb{P}(s(t+1) \mid s(t), s(t-1), \ldots, s(1)).$$

- The present state is a sufficient statistic of the future.

Andrey Markov(1856-1922)

Markov Process

A Markov process is a memory-less random process i.e. a sequence of random states $s(1), s(2), \ldots$ with the Markov property.

Definition

A Markov process (or a Markov chain) is a tuple $\langle \mathcal{S}, P \rangle$ *where*

- $\mathcal{S}$ *is a (finite) set of states, and*
- P *is a state transition probability function,*
  $P_{ss'} = \mathbb{P}[s(t+1) = s' \mid s(t) = s]$.

Markov Decision Process

A Markov decision process (MDP) is a Markov process with rewards and decisions.

Definition

A Markov decision process is a tuple $\langle \mathcal{S}, \mathcal{A}, R, P \rangle$ *where*

- $\mathcal{S}$ *is a (finite) set of states,*
- $\mathcal{A}$ *is a (finite) set of actions,*
- $R(s, a)$ *is a reward function,*
- P *is a state transition probability function,*
  $P^a_{ss'} = \mathbb{P}[s(t+1) = s' \mid s(t) = s, a(t) = a]$.

- Practical scenario : Learning to play chess.

Discounted-reward Markov Decision Process

A Markov decision process (MDP) is a Markov process with rewards and decisions.

Definition

A Markov decision process is a tuple $\langle \mathcal{S}, \mathcal{A}, R, P, \gamma \rangle$ *where*

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- P *is a state transition probability function,* $P^a_{ss'} = \mathbb{P}[s(t+1) = s' \mid s(t) = s, a(t) = a]$, *and*
- $\gamma \in (0, 1)$ *is a discount factor.*

- Practical scenario: Portfolio management. Why discounted? Distant reward not as valuable as immediate reward due to inflation.

Summary

- Introduction to reinforcement learning.
- Mathematical formulation of a RL problem.
- Formulating RL with multi-armed bandits and its variants.
- Formulating RL with Markov decision processes.

Next Lecture

- Simple solutions to bandits (and why they are sub-optimal?)
- An optimal solution : Upper confidence bound (UCB) algorithm.
- Proving the performance bound for UCB.

References

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